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CHAPTER 13

PROBABILITIES DISTRIBUTION

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RISK ASSESSMENT AND MANAGEMENT,
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HEALTHCARE AND EPIDEMIOLOGY

ABSTRACT

In this chapter we will study about basic probability distribution or theoretical distribution of the random variables both discrete and continuous. Individual and corporate generate several data that resemble certain theoretical distribution Scientifically there are several features of the theoretical distributions It needs to be prove for a quick analysis of this observe distribution.

Some illustration of theoretical distributions are

- Figure of female kids in a Kindred.
- Figure of defected goods in a factory
- Figure of candidates receiving salary in some limitations.
- The theoretical distributions are categorized into two categories--
- Discrete probability distribution
- Continuous probability distribution

13.1 CHARACTERISTICS OF PROBABILITY DISTRIBUTIONS

- **Mean ():** The expected value or average of the random variable:
  $\mu = \sum x P(X = x) \quad \text{(discrete)} \quad \text{or} \quad \mu = \int_{-\infty}^\infty x f(x) dx \quad \text{(continuous)}$
- **Variance ():** Measures the spread or dispersion of a distribution:

$$\sigma^2 = \sum (x - \mu)^2 P(X = x) \quad \text{(discrete)} \quad \text{or} \quad \sigma^2 = \int_{-\infty}^\infty (x - \mu)^2 f(x) dx \quad \text{(continuous)}$$

- **Standard Deviation ():** The square root of variance, representing the spread in the same units as the variable.

- **Skewness**: Indicates the asymmetry of a distribution. Positive skew implies a long tail on the right; negative skew implies a long tail on the left.

- **Kurtosis**: Measures the "tailedness" of the distribution. Higher kurtosis indicates more extreme outliers.

13.2 APPLICATIONS OF PROBABILITY DISTRIBUTIONS:

Probability distributions are a cornerstone of statistical analysis and mathematical modeling. Their ability to represent and analyze uncertainties makes them indispensable across numerous domains.
From predicting stock market trends to understanding disease outbreaks, probability distributions are used to describe random phenomena and make data-driven decisions. In this section, we explore the wide-ranging applications of probability distributions across various fields in detail.

13.2.1 BUSINESS AND FINANCE

Risk Assessment and Management

- Probability distributions help businesses and financial institutions quantify and manage risks.
- Credit Risk Modeling: Banks use probability distributions like the binomial and Poisson to estimate the likelihood of loan defaults.
- Operational Risks: Distributions such as Weibull and exponential are used to assess the likelihood and timing of system failures or other disruptions.

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13.2.2 STOCK MARKET ANALYSIS

- Price Movements: The normal distribution is often used to model stock returns due to its bell-shaped curve, assuming returns are symmetrically distributed around the mean.

- Volatility Estimation: Distributions like log-normal capture the asymmetric behavior of stock prices over time.

- Option Pricing: Financial models like the Black-Scholes formula use probability distributions to calculate the fair value of options.

13.2.3 DEMAND FORECASTING

Businesses use Poisson and normal distributions to forecast demand for products or services, helping optimize inventory and reduce wastage.

13.2.4 ENGINEERING AND MANUFACTURING

A. Quality Control

In manufacturing processes, binomial and normal distributions are commonly used to monitor quality. Example: A factory might use the binomial distribution to track the proportion of defective products in a batch.

13.2.5 RELIABILITY ENGINEERING

- Reliability analysis often relies on distributions to predict the lifespan and failure rates of components:

- Weibull Distribution: Used to model the time to failure of machines and components.

- Exponential Distribution: Applied to systems with a constant failure rate.

13.2.6 OPERATIONS RESEARCH

Queueing Theory: Probability distributions such as exponential and Poisson are used to analyze service systems, including waiting times in customer queues or server response times in networks.

13.2.7 HEALTHCARE AND EPIDEMIOLOGY

13.2.7.1 DISEASE MODELING

- Poisson distribution: Models the number of disease cases in a specific population over time.
- Normal Distribution: Used in analyzing test results such as blood pressure, cholesterol levels, and other health metrics.
- Exponential Distribution: Models the time between disease occurrences or the duration of a hospital stay.

13.2.7.2 SURVIVAL ANALYSIS

- Weibull Distribution: Widely used in medical studies to estimate patient survival times.
- Kaplan-Meier Analysis: Probability distributions are integral to survival curves for analyzing patient outcomes.

13.2.7.3 Drug Development

- Pharmaceutical companies use probability distributions to analyze clinical trial data:
- Beta Distribution: Used to model probabilities of success for new drugs.

- Normal Distribution: Assesses the efficacy of treatments by comparing treatment and control groups.

13.2.8 ENVIRONMENTAL SCIENCE

13.2.8.1 CLIMATE MODELING

Normal and Exponential Distributions: Used to analyze temperature, rainfall, and other weather phenomena.

Example: Meteorologists use these distributions to predict temperature ranges and rainfall patterns.

13.2.8.2 RISK ASSESSMENT FOR NATURAL DISASTERS

Extreme Value Distributions: Used to model rare events like floods, earthquakes, and hurricanes.

Example: Predicting the likelihood of a 100-year flood helps design resilient infrastructure.

13.2.9 SOCIAL SCIENCES AND PSYCHOLOGY

13.2.9.1 SURVEY ANALYSIS

Social scientists use discrete probability distributions like the binomial and multinomial to analyze survey results:

Example: Understanding the distribution of opinions across demographic groups

13.2.9.2 BEHAVIORAL STUDIES

Normal Distribution: Models the distribution of IQ scores and other psychological traits in the population.

Exponential Distribution: Models response times in cognitive experiments

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13.2.9.3 INCOME AND WEALTH DISTRIBUTIONS

Pareto Distribution: Often used to model the distribution of income and wealth, where a small percentage of the population controls a large share of resources.

Log-Normal Distribution: Captures the distribution of wages in a population.

13.2.10 COMPUTER SCIENCE

13.2.10.1 MACHINE LEARNING

Probability distributions play a key role in machine learning algorithms:
Gaussian Naive Bayes: Assumes features follow a normal distribution to classify data.

Hidden Markov Models: Use distributions like Poisson to model sequences, such as speech recognition or text processing.

13.2.10.2 RANDOM NUMBER GENERATION

Uniform Distribution: Generates random numbers for simulations, cryptography, and randomized algorithms.

13.2.10.3 NETWORK ANALYSIS

Erdős-Rényi Models: Use probability distributions to analyze random graphs and networks, such as social media interactions or traffic flows.

13.2.11 EDUCATION AND RESEARCH

13.2.11.1 TEST SCORE ANALYSIS

Normal Distribution: Models student performance on standardized tests, such as SAT or GRE scores.

Binomial Distribution: Used to analyze success rates in exams with binary outcomes (pass/fail).

13.2.11.2 EXPERIMENTAL DESIGN

Probability distributions guide hypothesis testing and data analysis in research:

T-Distributions: Compare means between groups in small sample studies.

Chi-Square Distribution: Tests goodness-of-fit for categorical data.

13.2.12 AGRICULTURE AND ENVIRONMENTAL STUDIES

13.2.12.1 CROP YIELD PREDICTIONS

Normal and Exponential Distributions: Used to predict crop yields under varying weather conditions.

13.2.12.2 PEST AND DISEASE SPREAD

Poisson distribution: Models the number of pests or disease outbreaks in a given area.

13.2.13 SPORTS AND GAMING

13.2.13.1 PLAYER PERFORMANCE

Binomial Distribution: Models outcomes such as the number of goals scored by a player in a game.

Normal Distribution: Evaluates player performance over a season.

13.2.13.2 GAME THEORY

Probability distributions are integral to analyzing strategies in competitive games, helping predict opponent behavior.

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13.2.14. DEFENSE AND AEROSPACE

13.2.14.1 MISSILE ACCURACY

Normal Distribution: Models the deviation of missile impacts from their intended targets.

13.2.14.2 RELIABILITY OF DEFENSE SYSTEMS

Weibull Distribution: Predicts the reliability and failure rates of defense equipment under different conditions.

13.2.15 ENERGY AND UTILITIES

13.2.15.1 POWER LOAD FORECASTING

Normal and Poisson Distributions: Used to predict electricity consumption and optimize grid operations.

13.2.15.2 RENEWABLE ENERGY ANALYSIS

Rayleigh Distribution: Models wind speeds for wind energy projects.

Weibull Distribution: Evaluates solar panel efficiency under different conditions.

13.2.16 TRANSPORTATION AND LOGISTICS

13.2.16.1 TRAFFIC FLOW ANALYSIS

Poisson distribution: Models the arrival of vehicles at intersections.

Normal Distribution: Used to analyze travel times and optimize routes.

13.2.16.2 SUPPLY CHAIN OPTIMIZATION

Exponential Distribution: Models delivery times and inventory replenishment.

13.2.16.3 REAL-LIFE EXAMPLES

- Weather Prediction: The probability of rainfall on a given day can be modeled using a binomial or normal distribution.

- Quality Control: In manufacturing, Poisson distributions can be used to model the number of defective products.

- Customer Behavior: Retailers use geometric distributions to analyze how many advertisements are needed to achieve a sale.

13.3 LIMITATIONS OF PROBABILITY DISTRIBUTIONS

Probability distributions provide powerful tools for modeling and analyzing random phenomena. They enable us to describe uncertainties, predict outcomes, and make informed decisions across disciplines such as business, engineering, healthcare, and social sciences. However, like any mathematical framework, probability distributions have limitations. These limitations arise from assumptions, real-world complexities, data quality issues, and challenges in application. This section explores the limitations of probability distributions in detail, offering insight into their practical constraints and the situations where they may fall short.

13.3.1 ASSUMPTIONS IN PROBABILITY MODELS

Probability distributions rely on a set of underlying assumptions. When these assumptions do not align with real-world conditions, the models may fail to represent the phenomena accurately.

13.3.1.1 INDEPENDENCE OF EVENTS

Many probability models, such as the binomial and Poisson distributions, assume that events are independent. In real-world scenarios, events often exhibit dependency.

Example: In social networks, one person's behavior (e.g., buying a product) may influence others, violating the independence assumption.

Impact: Using such models for dependent events can lead to inaccurate predictions.

13.3.1.2 HOMOGENEITY OF PARAMETERS

Probability distributions often assume that parameters like mean () or rate () remain constant.

Example: The Poisson distribution assumes a constant event rate over time. However, in traffic modeling, the rate of car arrivals may vary during peak and non-peak hours.

Impact: Failing to account for varying parameters leads to incorrect conclusions.

13.3.1.3. INFINITE SAMPLE SPACES

Continuous distributions like the normal or exponential assume an infinite range of values, which may not be practical.

Example: Heights of individuals follow a normal distribution but cannot realistically be negative or exceed certain biological limits.

Impact: Such assumptions can produce nonsensical predictions at extremes.

13.3.2 DATA QUALITY ISSUES

The accuracy of probability distributions depends heavily on the quality and quantity of data available. Poor data can limit the effectiveness of probability models.

13.3.2.1 INSUFFICIENT DATA

Small datasets or sparse data can make it difficult to estimate distribution parameters accurately.

Example: Estimating the mean and variance of a population from a small sample may lead to overfitting or under fitting.
Impact: Limited data may result in unreliable predictions or invalid conclusions.

13.3.2.2. MEASUREMENT ERRORS

Data collection often involves errors due to faulty instruments, human mistakes, or missing values.

Example: In a manufacturing process, sensors may record incorrect measurements due to calibration issues.

Impact: Such errors distort the probability distribution, reducing its reliability.

13.3.2.3. OUTLIERS

Real-world datasets often contain extreme outliers that deviate significantly from the rest of the data.

Example: In income distributions, a few individuals may have exceptionally high incomes, skewing the results.

Impact: Outliers can heavily influence parameters like the mean and variance, leading to inaccurate models.

13.3.3 MISREPRESENTATION OF COMPLEX REAL-WORLD PHENOMENA

Real-world phenomena are often more complex than the assumptions underlying standard probability distributions.

13.3.3.1 MULTIVARIATE DEPENDENCIES

Many real-world problems involve multiple variables that interact in complex ways.

Example: In weather forecasting, temperature, humidity, and pressure are interdependent.

Impact: Standard univariate distributions (e.g., normal or Poisson) cannot capture these dependencies, requiring multivariate or specialized models.

13.3.3.2 NON-STATIONARITY

Some processes change over time or space, violating the stationary assumptions of many distributions.

Example: Stock market returns may exhibit varying volatility across different time periods.

Impact: Static distributions fail to account for such dynamics, necessitating time-series models.

13.3.3.3 NON-NORMALITY

Many real-world datasets do not follow common distributions like the normal distribution.

Example: Financial returns often exhibit heavy tails and skewness, making them unsuitable for normal distribution modeling.

Impact: Applying inappropriate distributions leads to incorrect results and poor decision-making.

13.3.4 CHALLENGES IN PARAMETER ESTIMATION

Parameter estimation is crucial for defining probability distributions. However, it can be challenging due to the following reasons:

13.3.4.1 OVERFITTING

In cases with limited data, fitting a complex distribution to match the dataset may result in overfitting.

Example: Using a high-order polynomial to model a dataset results in a curve that fits the data perfectly but lacks generalization.

Impact: Overfitted models fail to predict new data accurately.

13.3.4.2 UNDERFITTING

Conversely, using overly simplistic models may fail to capture important patterns in the data.

Example: Fitting a linear model to a dataset with nonlinear trends.

Impact: Underfitted models provide poor approximations, reducing their predictive power.

13.3.4.3 COMPUTATIONAL COMPLEXITY

Estimating parameters for complex distributions (e.g., multivariate distributions) may involve computationally intensive methods such as maximum likelihood estimation (MLE) or Bayesian inference.

Example: Estimating parameters for a mixture of Gaussian distributions can be time-consuming and resource-intensive.

Impact: Computational demands may limit the applicability of certain distributions in large-scale or real-time scenarios.

13.3.5 SENSITIVITY TO OUTLIERS AND EXTREME VALUES

Probability distributions, especially those relying on the mean and variance, can be highly sensitive to outliers and extreme values.

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Example: Normal Distribution

The normal distribution assumes symmetry and is heavily influenced by extreme values, which can shift the mean and inflate the variance.
In financial modeling, rare events (e.g., economic crashes) may be underestimated by the normal distribution.

Impact
Misrepresenting the true nature of data, particularly in high-stakes fields like risk management.

13.3.6. INAPPLICABILITY IN NON-QUANTIFIABLE SCENARIOS

Probability distributions are inherently quantitative and cannot be applied to qualitative data without proper encoding.

Example: Survey Data

Survey responses like "Strongly Agree" or "Neutral" are categorical and cannot be directly modeled using probability distributions without converting them into numerical values.

Impact: The need for data preprocessing introduces potential biases and inaccuracies.

13.3.7. OVERRELIANCE ON SIMPLISTIC MODELS

While simple distributions like normal or Poisson are easy to use, they may not always be suitable for complex phenomena.

Example: Financial Markets

Financial returns often exhibit fat tails (extreme events) and volatility clustering, which standard distributions fail to capture.

Impact: Overreliance on simplistic models like the normal distribution can lead to catastrophic errors, such as underestimating the likelihood of financial crises.

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13.3.8 MISINTERPRETATION AND MISUSE

Probability distributions are sometimes misused or misinterpreted due to a lack of understanding.

13.3.8.1 CONFUSING CORRELATION WITH CAUSATION

Probabilities indicate relationships between variables but do not establish causation.

Example: A correlation between ice cream sales and drowning incidents does not imply causation.

Impact: Misinterpretation can lead to flawed decision-making.

13.3.8.2 MISREPRESENTING UNCERTAINTY

A probability distribution provides a model of uncertainty but does not eliminate it.

Example: Predicting a 90% chance of rain does not guarantee rainfall.

Impact: Misrepresenting probability as certainty can erode trust in models.

13.3.9 LIMITED FLEXIBILITY FOR RARE EVENTS

Most standard distributions fail to capture the occurrence of rare but impactful events, such as natural disasters or economic crashes.

Example: Black Swan Events

Events with low probabilities but high impacts are poorly modeled by distributions like the normal distribution.

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Impact: This limitation can lead to underestimating risks, especially in domains like finance and disaster management.

13.3.10 THEORETICAL VS. EMPIRICAL DISTRIBUTIONS

Probability distributions are often theoretical constructs, while real-world data may not conform to these theoretical shapes.

13.3.10.1 THEORETICAL MODELS

Idealized distributions like the normal or exponential are based on mathematical assumptions.

Impact: Real-world data often deviates due to noise, biases, or other factors.

13.3.10.2 EMPIRICAL DISTRIBUTIONS

Empirical data may not align perfectly with any standard distribution.

Example: Income distributions often exhibit heavy skewness and fat tails, which standard distributions cannot capture.

13.4 RANDOM VARIABLE

Random variable is a variable which calculates the occurrence of events.

13.4.1 DISCRETE RANDOM VARIABLE

Discrete random variable is used when favorable outcomes in a probability calculations are counted when a unbiased coin flipping one time probability event is 2, twist of fate when the unbiased Coin tossed two times probability event is 4 and in the above 2 example the number of outcomes are definite and is known as Discrete random variable.

Examples:
Number of heads in 10 coin tosses (0, 1, 2, ..., 10).
Number of defective items in a batch (0, 1, 2, ...).

Key Properties:

- Takes specific, separate values (e.g., integers).
- Probabilities are assigned to each value, and the total probability is 1.
- Modeled by distributions like Binomial, Poisson, or Geometric.

13.5 PROBABILITY MASS FUNCTION (PMF):

The probability of a discrete random variable being equal to a specific value is:
$P(X = x)$

13.5.1 CONTINUOUS RANDOM VARIABLE

Continuous chance variable is used when the number of reactions in a probability calculations is innumerable for example the values of bus timings for exit and entrance at a bus stop are the Continuous random variables. Just like weight and the IQ level quotient of the People.

Examples:
Heights of students (e.g., 160.5 cm, 162.3 cm).
Time taken to complete a task (e.g., 2.4 hours, 3.8 hours).

Key Properties:

- Takes infinite values within a range (e.g., all real numbers between 0 and 1).
- Probabilities are defined over intervals, not specific points, because the probability at a single point is 0.
- Modeled by distributions like Normal, Uniform, or Exponential.

13.6 PROBABILITY DENSITY FUNCTION (PDF)

For continuous random variables, probabilities are represented by a PDF. The probability that lies within an interval is:
P(a \leq X \leq b) = \int_a^b f(x) \, dx

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13.6.1 PROBABILITY DISTRIBUTIONS

A probability distribution describes how the probabilities of different outcomes are distributed for a random variable. It provides a mathematical function or table that specifies the likelihood of each possible outcome.
Probability distributions are categorized into two categories discrete and continuous.

13.6.2 DISCRETE PROBABILITY DISTRIBUTION

For variables with specific values (e.g., rolling a die).
Example: Binomial, Poisson.
With, the help of binomial distribution and poission distribution through which we can solve any hypothetical into conceptual probability.

13.6.3 CONTINOUS PROBABILITY DISTRIBUTIONS

For variables that take on an infinite number of values within a range (e.g., heights of people).
Example: Normal, Exponential.
Continuous probability distribution is a random variable in which it assumes the value with the help of normal distribution table.

13.7 BIONOMIAL DISTRIBUTION

Binomial distribution is a variable which can be solve with the help of P,Q, n where P is favorable outcome and Q is non-favorable outcome and n is total outcome .
In this theorem formula is $(a+b)^n = \sum^n_{r=0} {}^nC_r\, a^{n-r} b^r$, where n is a positive integer and a, b are real numbers, and $0 < r \leq n$.
$P(x) = {}^nC_x \cdot p^x (1 - p)^{n-x}$

<u>Properties:</u>

1. Fixed number of trials (n): The process consists of a specific number of trials.

2. Independent trials: The outcome of one trial does not affect another

.3. Two outcomes per trial: Each trial has two possible outcomes — success (with probability p) or failure (with probability 1-p).

4. Constant probability: The probability of success (p) remains the same for all trials.

13.7.1 ASSUMPTION FOR APPLYING A BINOMIAL DISTRIBUTION

These are some cases where we apply binomial distribution -

- The binomial experiment is P,Q Base where P is favorable and Q is non-favorable
- The probability of favorable result must be equal for all hit when a coin tossed two or three time getting a head is always 0.5 and tail are also 0.5.
- It needs to be conducted under similar conditions.

Examples of binomial variate:

There are some examples of binomial variate –

- From manufactured lot 6 articles are drawn for defective random sample
- when a coin is flipped 8 times number of any front turn

Illustration 1

An impartial coin is flipped six chance. What is the chance that Result in 2 heads Minimum 5 heads Up to 2 heads up to 1 head more than 5 heads Minimum 1 head

Answer

Let 'A' getting heads .

$p = ½ , q = ½ , n = 6$

Therefore, by binomial distribution,

$P(X = x) = {}^6C_x(1/2)^{6-x}(1/2)^6$

¡) It needs to be calculate getting two heads

$P(X = 2) = {}^6C_2(1/2)^{6-2}(1/2)^6$

$= (6/1×5/2)(1/2^4)(1/2^2)$

$= 15/64$

Therefore answer of two Heads is 15/64.

¡¡) The probability minimum 5 heads

$P\{X \geq 5) = P(X = 5) + P(X = 6)$

$= {}^6C_5(1/2)^{6-5(}1/2)^5 + {}^6C_6(1/2)^{6-6}(1/2)^6$

$= 6*(1/2)^6+(1/2)^6$
$= 7/64$
Therefore, the probability minimum is 5 heads
Heads is 7/64.

iii) The probability up to 2 heads is givenby:
$P(X \leq 2) = P(X=0)+P(X=1)+P(X=2)$
$=(1/2)^6+{}^6C_1(1/2)^{6-1}(1/2)^6+{}^6C_2(1/2)^{6-2}(1/2)^6$
$P(X \leq 2)= (1/64)+ (6\times1/64)+ (6/1 \times 5/2 * 1/64)$
$= 1/64+ 6/64+ 15/64$
$= 22/64$
$= 11/32$
Therefore, the probability up to twoHeads is 11/32.
iv) The probability up to one head
$P(X \leq 1) = P(X =0)+ P(X = 1)$
$= 1/64+ 6/64$
$= 7/64$
Therefore, the probability up to 1 head is 7/64.

v) The probability that more than 5 heads
$P(X \geq 5) = P(X = 5)+ P(X = 6)$
$= 6/2^6+ 1/2^6$
$= 7/64$
Therefore, the probability more than fiveHeads is 7/64.

vi)The probability minimum one head is given by:
$P(X\geq1)=1-P(X<1)$
$=1-P(X=0)$
$=1-1/2^6$
$= 1-1/64$
$= 63/64$
Therefore, the probability minimum one headis 63/64.ssss

Solved Problem 2

In a cement factory disease chances is 20% therefore there are five employe

Calculate probability
None contact the disease
Two contact the disease
Minimum contract the disease

Ans . p = 20/100 = 0.2

$\therefore$ q = 1- 0.2 = 0.8

n = 5

such that ,

$P(X = x)= {}^5Cx(0.8)^{5-x}(0.2)^x$

i)The probability that none contact the disease is given by:

$P(X=0)=(0.8)^5 = 0.3277$

Such that, the probability that none contact the disease is 0.3277.

ii) The probability that 2 contact disease is given by:

$P(X=2)= {}^3C_2 (0.8)^3 (0.2)^2$

$=(10)(0.512)(0.04)=0.2048$

Therefore, the probability that two contact disease is 0.2048.

iii) The probability that minimum 4 contact the disease .

$P(X > 4)= P(X = 5)= (0.2)^5 = 0.00032$

Therefore, the probability that minimum contact the disease is 0.00032.

13.7.2 POISSON DISTRIBUTION

Poisson distribution apply when number of trials selected in a large number like thousand two thousand and four thousand and chance is 2, 3, 4 then we calculate mean m is equal to np where p is the favorable chance. In this case binomial distribution cannot be applicable

$P(X) = e^{-m} m^x/\angle x$

Characteristics:

- Discrete Distribution: It deals with the count of events (e.g., number of customer arrivals).
- Independent Events: Occurrence of one event does not affect another.

- Constant Rate (λ): The average number of events per interval (λ) is constant.
- No Overlapping Events: Two events cannot happen simultaneously.

Poisson distribution assumptions

- The outcome of trial must be bifurcated
- success must remain similar
- The trials should be statistically independent.

Example-There are 2000 houses in town. Chances of catching fire is 2 in one thousand house what is the probability that:

- No houses catches fire
- minimum one house catch fire

Solution

P=2\1000=0.002 and n=2000

M=np=4

The probability that no house catches fire

$P(X) = e^{-m} m^{x}/\angle x$

P(0)=0.01832

The probability that minimum house catches fire

P(X>1)=1-P(X=0)s

1-0.01832

=0.98168.

13.7.3 NORMAL DISTRIBUTION

The quantitative variables which consist the measure of height, weight humidity and so on are example

Normal Distribution features

- It is continuous distribution
- It mean is μ and standard division is σ where μ and σ are the parameters of the distribution
- It is well saved figure and is proper about its mean.
- The mean line divides two equal part

LIMITS	AREA%

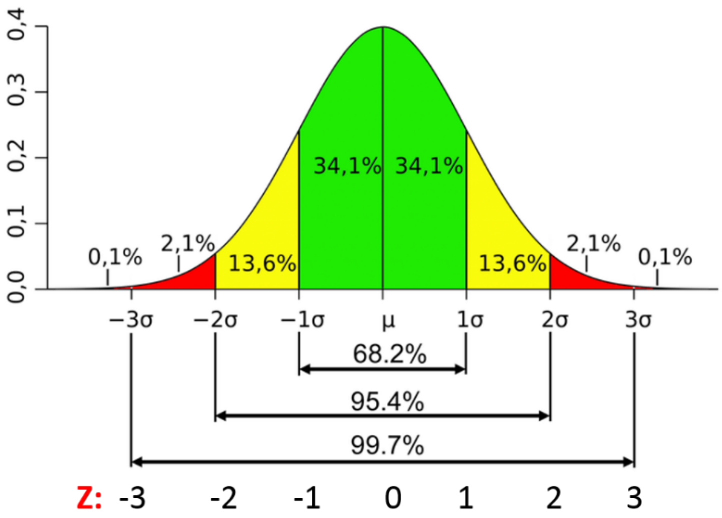
μ±σ	68.2
μ±1`96σ	95
μ±2σ	95.4
μ±3 σ	99.7

Understanding Normal Distribution

The normal distribution is very popular distribution it represents a well saved curve and its mean line is divided into equal part 50% left and 50% right Left represent less than mean right part represent more than mean .

The Formula for the Normal Distribution

Z=X-μ/σ



S.N.D

Example no. 1 – The weight of bourn vita the follows normal distribution his mean weight is 500gm and standard division 10gm if select one packet what will be the probability.

1. The packs weight will be greater than 515 gms ?

. The packs weight will be between

480 to 500 gms ?

The packet > 480 and < 520

If 10000 packets are delivered, how many packets will be cancelled , In 480 gms and 520 gms for ?

Solution: by normal curve--

It will exceed 515 gms

$P(X \geq 515) = 0.5 - P(500 \leq X \leq 515)$

$= 0.5 - P(500-500/10 \leq Z \leq 515-500/10) = 0.5 – P(0 \leq Z \leq 1.5) = 0.5 – 0.4332 = 0.668$

Therefore, answer 515 gms is 0.0668.

2-The probability ,waight lie within 480 to 520 gms

P[480< x<< 520=P(480<x< 500) +P (480 < x <500

.4772+.4772=.9544

$P(480 \leq X \leq 520) = 0.9544$

If the weight lies outside these values then it will be rejected.

:. The probability of rejection =1-0.9544=0.0456

The number of packets that will be rejected is given by N × P.

N×P = 10000 x 0.0456 = 456

The rejected packet will be 456.

Example no 2

X is a Normal variate mean 42 and standard deviation 4. calculate the Probability of value X

Less than 50

Greater than 50

Solution

X is a normal variate .

$\mu = 42$ and o = 4.

Therefore,

$Z = X - \mu/4$

$= X - 42/4$ Is a Standard normal variate.

$P(X < 50) = P(X - 42/4 < 50 - 42/4) = P(Z < 2)$

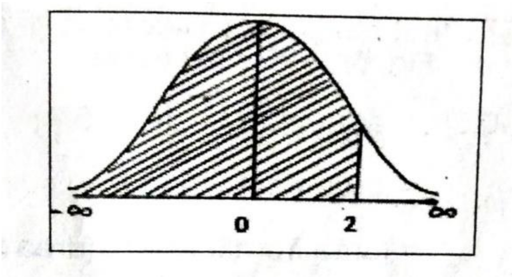


Fig. 1: Normal curve

P (Z< 2)= Standard normal area from - ∞ to 2

= [area from ∞ to 0] + [area from 0 to 2]

= 0.5 +0.4772 (from the table)

= 0.9772.

P (X < 50) = P(X- 42/4 < 50- 42/4) = P (Z > 2)

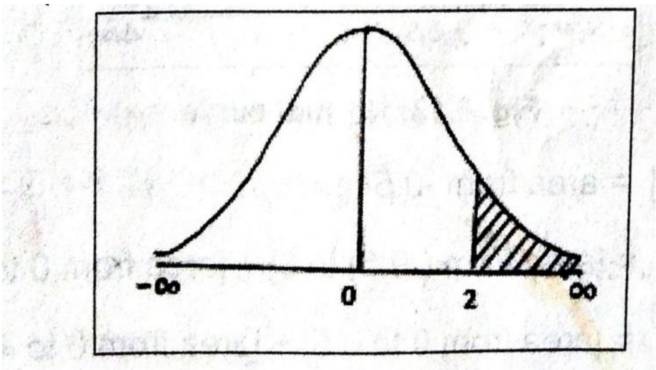


Fig. 2: Normal curve

P(Z > 2)= area from 2 to ∞

= [area from 0 to ∞] – [area from 0 to 2]

= 0.5-0.4772 (from the table)

= 0.0228.

Example-3

Mean life of fan construct by a unit is 1200 hours. The standard deviation is 200 hours.

In 10,000 fans, how many fan life1050 hours or more?

2) What is the % of fans which are look for fail before 1050hours of favour ?

Solution

Let X' denote the life of the fans. Then, X is a normal variate with

Parameters $\mu = 1200$hrs and $\sigma = 200$hrs

$Z = X-\mu \div \sigma$

$Z = X-1200 \div 200$ is a Standard normal variate.

Probability that life of a fan is 1050 hours or more is

$P(X \geq 1050) = P[X-1200 \div 200 \geq 1050-1200 \div 200] = P[Z \geq -0.75]$

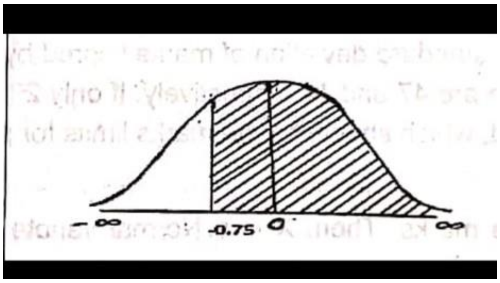


Fig 3: Normal curve

P[Z≥-0.75] = [area from -0.75 to 0] + [area from 0 to ~]

= [area from 0 to 0.75] + [area from 0 to ∞]

=0.2734 + 0.5 = 0.7734

(since normal distribution is symmetrical -0.75 value is same as 0.75)

In a lot of N = 10,000 fans , Await number of fans with life 1080 Hours or more is N x P [X21050] = 10,000 x 0.7734 = 7734

Probability that life of a fan is 1050 hours or less is

$P(X \geq 1050) = P[X-1200 \div 200 > 1050-1200 \div 200] = P[Z \geq -0.75]$

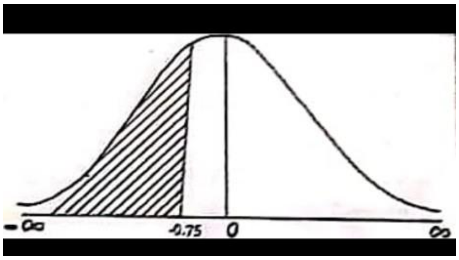


Fig 4: Normal curve

P(Z ≤ -0.75) =[area from 0 to] – [area from 0 to 0.75)

= 0.5-0.2734 = 0.2266

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The % of fan age < 1500 hours is
100 x P(X ≤1050) = 100 x 0.2266 = 22.66

13.8 CONCLUSION

Probability distributions are a cornerstone of mathematics and statistics, providing a structured framework for understanding and modeling randomness. Their significance lies in their ability to represent the variability inherent in natural and human-made systems, transforming uncertainty into quantifiable and analyzable forms. By assigning probabilities to different outcomes, probability distributions enable researchers, professionals, and decision-makers to gain valuable insights into random phenomena, make informed predictions, and address complex challenges across a wide array of fields.

One of the most significant contributions of probability distributions is their role in simplifying complex systems. Real-world phenomena, such as stock price movements, machine failures, disease outbreaks, and weather patterns, often involve uncertainty and randomness that cannot be fully explained by deterministic models. Probability distributions provide a mechanism to approximate these uncertainties and predict their behavior. For instance, the normal distribution, with its symmetric bell curve, models numerous natural and social processes, while the Poisson distribution captures the occurrence of rare events like accidents or system failures. Such distributions form the basis for statistical inference, helping practitioners evaluate hypotheses, test models, and draw conclusions with a clear understanding of associated uncertainties.

The applications of probability distributions are as diverse as they are profound. In finance, they are used to estimate risks, analyze market trends, and price financial instruments. In healthcare, they are central to understanding disease progression, predicting survival rates, and evaluating drug efficacy. Engineering relies on probability distributions for quality control, reliability testing, and system design, while fields like environmental science use them to model weather patterns, predict natural disasters, and assess climate risks. Social scientists use probability distributions to analyze survey data, study income distributions, and explore behavioral patterns. Their universality ensures their relevance across virtually every domain of human activity.

Despite their widespread utility, probability distributions are not without limitations. Many rely on assumptions—such as data normality, independence, or stationarity—that may not always hold in real-world settings. When these assumptions are violated, the accuracy and reliability of predictions can be compromised. Additionally, the quality of data plays a critical role in the effectiveness of probability distributions. Poor or incomplete datasets can lead to distorted models, and misinterpretations of results can have serious consequences, especially in fields like finance, healthcare, and public policy. Furthermore, while traditional distributions are effective for many applications, they often struggle to model complex phenomena involving extreme values, multivariate dependencies, or dynamic changes over time.

As we move into a data-driven era, the relevance of probability distributions continues to grow. Emerging technologies, such as machine learning and artificial intelligence, are integrating probability distributions into predictive algorithms, enabling models to adapt to complex, high-dimensional datasets. Innovations like dynamic and non-stationary models are addressing traditional limitations, improving the ability to handle evolving systems. The integration of traditional probability theory with advanced computational tools offers new opportunities to model and analyze phenomena that were previously considered intractable.

In conclusion, probability distributions are more than just mathematical abstractions; they are indispensable tools for understanding and navigating uncertainty. Their ability to describe randomness, predict outcomes, and inform decision-making has made them fundamental to science, technology, and human progress. As their methodologies evolve, probability distributions will remain at the forefront of tackling the uncertainties of an increasingly complex and interconnected world.

13.9 REFERENCE

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